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## NURBS skinning surface for ship hull design based on new parameterization method

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**Abstract** Surface reconstruction from sets of cross-sectional data is important in a variety of applications. The problem of generating a ship hull surface from non-regular cross-sectional curves is addressed. Generating non-uniform rational B-splines (NURBS) surfaces that represent cross-sectional curves is a challenge, since the number of control points is growing due to the non-avoidable process of having compatible cross-sectional curves. A new NURBS parameterization method that yields a minimum number of control points, and is adequate in generating a smooth and fair NURBS surface for ship hulls is proposed. This method allows for multiple knots and close domain knots. The results of applying different parameterization methods on the forward perpendicular (FP) region of a ship hull (organized in eight sections) shows that the proposed method reduces the number of control points and generates a smooth and fair NURBS surface, without sacrificing the original object shape of the FP region.

**Keywords** Compatibility process · Non-uniform rational B-splines (NURBS) skinning · Parameterization · Skinning process

### 1 Introduction

Non-uniform rational B-splines (NURBS) surfaces have become a de facto standard in surface designation and representation because of their powerful geometrical properties and ability to

represent both freeform and analytical shapes [1–3]. One of the more interesting NURBS applications is the skinning surface method, which is applicable to a ship's hull since it is usually organized in the form of cross-sectional curves. Here, we address the problem of NURBS skinning surface approximation to a set of cross-sectional curves, where the number of control points varies from section to section. The skinning process can deal with open or close sections. Many researchers have discussed this problem in general terms [4–8]. The main problem of the skinning process is to have compatible cross-sectional curves. This process can be carried out by a traditional skinning method [2, 9]. Although these approaches are straightforward, they end up with a large number of control points in the resultant skinning surface. This is mainly due to the process of progressively merging different knot vectors to have compatible cross-sectional curves defined on a common knot vector. And it becomes more significant when the number of cross-sectional curves increase and the number of control points varies from section to section.

There a number of approaches that have been designed to reduce the number of control points without sacrificing with the original shape of cross-sectional curves [6, 7, 10–12]. Although some of the approaches reduce the control points of the resultant skinning surface, there is room for more reduction.

In this paper, we develop an approach to NURBS skinning surface similar to [11] to deal with open cross-sectional curves with a varying number of control points. This approach is based on a proposed parameterization method that makes use of the rational/irrational B-spline basis functions where for each span index, there is exactly one maximum value of rational/irrational B-spline basis functions. It also applies the centripetal parameterization method. An initial common knot vector is selected by specifying the cross-sectional curve with the highest number of control points, and the process of knot removal is applied to minimize the number of control points of this section without sacrificing the curve section shape. The resultant knot vector is selected as a common knot vector that is used for the compatibility process. The number of control points in each cross-sectional curve is less than (in the worst case equal to) the highest num-

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ber of control points, so that the approach provides a significant reduction of data in the resulting skinning surface while maintaining the original object shape of the cross-sectional curves.

This paper is organized as follows. In Sect. 2, NURBS skinning formulations are briefly introduced. Section 3 introduces the procedures of knot removal and knot refinement for the compatibility process. Section 4, introduces the developed skinning approach and the proposed parameterization method. In Sect. 5, the results of applying our approach based on different parameterization methods are presented and discussed. In Sect. 6, we offer conclusions.

## 2 NURBS skinning formulations

Skinning is the process of passing a surface through a given set of  $K$  rational cross-sectional curves  $C_k^w(u)$ ,  $k = 0, 1, \dots, K$ . So for convenience, NURBS curves and surfaces are presented below [2].

A NURBS curve of degree  $p$  is given by:

$$C_k^w(u) = \sum_{i=0}^n N_{i,p}(u) P_i^w, \quad (1)$$

where  $P_i^w = (w_i x_i, w_i y_i, w_i z_i, w_i)$  are the weighted control points, and  $N_i^w(u)$ ,  $i = 0, 1, \dots, n$  are the B-spline basis functions. The NURBS curve can be written in the rational form as follows:

$$C_k(u) = \sum_{i=0}^n R_{i,p}(u) P_i, \quad (2)$$

where  $P_i = (x_i, y_i, z_i)$  are 3D control points, and  $R_{i,p}(u)$ ,  $i = 0, 1, \dots, n$  are the rational B-spline basis functions, which are defined as:

$$R_{i,p}(u) = \frac{N_{i,p}(u) w_i}{\sum_{l=0}^n N_{l,p}(u) w_l}, \quad i = 0, 1, \dots, n, \quad (3)$$

where  $w_l$  is the weight associated with each control point  $P_l$ .

Both the rational/irrational B-spline basis functions  $R_{i,p}(u)$  and  $N_{i,p}(u)$  are defined over the knot vector as:

$$U = \{a, \dots, a, u_{p+1}, \dots, u_n, b, \dots, b\}.$$

Similarly, a NURBS surface of degree  $p \times q$  is given by:

$$S^w(u, v) = \sum_{i=0}^n \sum_{j=0}^m N_{i,p}(u) N_{j,q}(v) P_{i,j}^w, \quad (4)$$

where  $P_{i,j}^w = (w_{i,j} x_{i,j}, w_{i,j} y_{i,j}, w_{i,j} z_{i,j}, w_{i,j})$  (the control net of the weighted control points), and  $N_{i,p}(u)$  and  $N_{j,q}(v)$ ,  $i = 0, 1, \dots, n$ ;  $j = 0, 1, \dots, m$  are the B-spline basis functions. Equation 4 can be written in the rational form as follows:

$$S(u, v) = \sum_{i=0}^n \sum_{j=0}^m R_{i,j}(u, v) P_{i,j}, \quad (5)$$

where  $P_{i,j} = (x_{i,j}, y_{i,j}, z_{i,j})$  are 3D control points, and  $R_{i,j}(u, v)$

are the rational B-spline basis functions, which are defined as:

$$R_{i,j}(u, v) = \frac{N_{i,p}(u) N_{j,q}(v) w_{i,j}}{\sum_{k=0}^n \sum_{l=0}^m N_{k,p}(u) N_{l,q}(v) w_{k,l}}, \quad (6)$$

$$i = 0, 1, \dots, n; \quad j = 0, 1, \dots, m.$$

The B-spline basis functions  $N_{i,p}(u)$  and  $N_{i,p}(u)$ , and the rational B-spline basis functions  $R_{i,j}(u, v)$  are defined over the knot vectors as:

$$U = \{a, \dots, a, u_{p+1}, \dots, u_n, b, \dots, b\}$$

$$V = \{a, \dots, a, v_{q+1}, \dots, v_m, b, \dots, b\},$$

with ends  $(p+1)$  and  $(q+1)$ , and multiple knots of  $U$  and  $V$ , respectively. In most applications, the first  $(p+1)$  and  $(q+1)$  knots are assigned "0", and the last  $(p+1)$  and  $(q+1)$  knots are assigned "1". Throughout this paper the degrees are assumed to be bicubic ( $p = q = 3$ ), the first  $(p+1)$  and  $(q+1)$  knots are assigned 0 (i.e.  $a = 0$ ), and the last  $(p+1)$  and  $(q+1)$  knots are assigned 1 (i.e.  $b = 1$ ).

A NURBS skinning surface for a set of data points in terms of rational cross-sectional curves ( $K$  cross-sections)  $C_k^w(u)$ ,  $k = 0, 1, \dots$ , with  $K$  defined over the knot vectors  $U^k = \{u_0^k, u_1^k, \dots, u_{n_k}^k\}$ , is the surface that contains all the rational cross-sectional curves and is given by  $S^w(u, v)$ . There is no guarantee that cross-sections will have the same number of control points. As such, the compatibility process makes compatible cross-sections by merging the set of the knot vectors  $U^k$ . This means increasing the number of control points. In our paper, the compatibility process will be performed using a knot removal algorithm to reduce the number of control points for the cross-sections with the most control points; and a knot refinement algorithm to increase the number of control points for the sections with the smallest number of control points. These will be discussed in the next sections.

## 3 Compatibility process

To begin the compatibility process for the  $K$  rational cross-sectional curves in the skinning process, knot removal and knot refinement algorithms are presented below.

For any rational cross-sectional curve,  $C^w(u)$ , which has the largest number of control points defined over the knot vector  $U = \{0, \dots, 0, u_{p+1}, u_{p+2}, \dots, u_n, 1, \dots, 1\}$ , is removed without sacrificing the shape of the cross-sectional curve; or, more than one knot of the domain knots will be removed with respect to error tolerance [13–17]. Due to the removal of knot(s), it is necessary to compute new control points on the NURBS curve that represent the cross-sectional curve. The  $i$ th control points are computed (due to the removal of the knot  $u = u_r \neq u_{r+1}$ ) as follows [2]:

$$P_i^1 = \frac{P_i^0 - (1 - \alpha_i) P_{i-1}^1}{\alpha_i}, \quad r - p \leq i \leq 0.5(2r - p - 2)$$

$$P_j^1 = \frac{P_j^0 - \alpha_j P_{j+1}^1}{1 - \alpha_j}, \quad 0.5(2r - p + 1) \leq j \leq r - 1. \quad (7)$$

This can be generalized to handle the removal of multiple knots  $u = u_r$  (of multiplicity  $s$ ),  $t$  times as follows:

$$P_i^t = \frac{P_i^{t-1} - (1 - \alpha_i) P_{i-1}^t}{\alpha_i},$$

$$0.5(2r - p - s + t + 1) \leq j \leq r - s + t - 1,$$

$$P_j^t = \frac{P_j^{t-1} - \alpha_j P_{j+1}^t}{1 - \alpha_j}, \quad (8)$$

$$0.5(2r - p - s + t + 1) \leq j \leq r - s + t - 1,$$

where  $\alpha_i = \frac{u - u_i}{u_{i+p+t} - u_i}$ , and

$$\alpha_j = \frac{u - u_{j-t+1}}{u_{j+p+1} - u_{j-t+1}}. \quad (9)$$

On the other hand, for any of the cross-sectional curves  $C^w(u)$  with the smallest number of control points, knot refinement is applied to raise the number of control points [2, 18–22]. For the section of the curve  $C^w(u)$  with the control points  $P_i^w$ , the insertion of knot(s) yields new control points  $Q_i^w$ . This proceeds as follows:

Let

$$C^w(u) = \sum_{i=0}^n N_{i,p}(u) P_i^w, \quad (10)$$

be a NURBS curve defined as the knot vector  $U = \{0, \dots, 0, u_{p+1}, u_{p+2}, \dots, u_n, 1, \dots, 1\}$ . Inserting a new knot  $\bar{u} \in [u_k, u_{k+1})$  into the knot vector  $\bar{U}$ ,  $C^w(u)$  can be represented in the form:

$$C^w(u) = \sum_{i=0}^{n+1} \bar{N}_{i,p}(u) Q_i^w, \quad (11)$$

where  $\bar{N}_{i,p}(u)$  are the  $p$ th-degree basis functions defined on  $\bar{U}$ . Thus,

$$\sum_{i=0}^n N_{i,p}(u) P_i^w = \sum_{i=0}^{n+1} \bar{N}_{i,p}(u) Q_i^w. \quad (12)$$

Equation 12 performs a series of  $(n + 2)$  linear equations that can be solved to obtain the control points  $Q_i^w$ . The easiest way to obtain  $Q_i^w$  is by applying the property “in any given knot span  $[u_k, u_{k+1})$ , at most  $p + 1$  of the  $N_{i,p}(u)$  are non zero, namely the functions  $N_{k-i,p}(u), N_{k-i+1,p}(u), \dots, N_{k,p}(u)$ ”. Thus,  $Q_i^w$  can be obtained as follows:

$$Q_i^w = \alpha_i P_i^w + (1 - \alpha_i) P_{i-1}^w, \quad (13)$$

where

$$\alpha_i = \begin{cases} 1 & i \leq k - p, \\ \frac{\bar{u} - u_i}{u_{i+p} - u_i} & k - p + 1 \leq i \leq k, \\ 0 & i \geq k + 1, \end{cases} \quad (14)$$

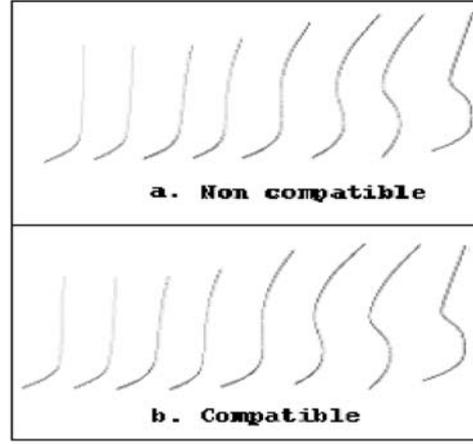


Fig. 1. A cubic NURBS curve

which can be generalized to handle the insertion of many knots at once. The result of cubic NURBS curves that represent the cross-sectional curves after applying both knot removal/refinement algorithms are shown in Fig. 1. The remaining part of the compatibility process will be discussed in the following section.

#### 4 NURBS skinning surface

To accomplish the compatibility process, skinning methods will be discussed. The conventional skinning approaches under our assumption can be summarized as follows [2, 9]:

- Make the rational cross-sectional curves compatible or defined over a common knot vector by merging the individual knot vectors, as shown in Sect. 3 above.
- Passing a NURBS surface through the compatible rational cross-sectional curves to generate the desired surface.

Although these approaches seem to be easy, a large number of control points are expected in the resultant skinning surface. Recently, [11, 12] developed approaches to reduce the number of the control points in the skinning surface, but there is still room to have more data reduction. Thus, we propose our method which based on parameterization and makes use of a knot removal algorithm, as well as a knot refinement algorithm. Our method is summarized in the following subsection.

##### 4.1 Developed skinning approach

The approach can be summarized in the following steps:

- Assume  $K$  rational cross-sectional curves  $C_k^w$  and degrees  $p = q = 3$ .
- Find  $\hat{n}$ , the highest index for the cross-sectional curve(s) with largest number of control points [11], compute the averaging parameter knot vector  $U_{\hat{n}}$  based on the proposed parameterization method (see Sect. 4.2).
- Use  $U_{\hat{n}}$  and the values of the proposed parameterization to calculate the control points on the NURBS curve  $C_k^w(u)$ .
- Apply knot the removal algorithm to the knot vector  $U_{\hat{n}}$ , as shown in Sect. 3, to achieve the minimum number of control

- points on the NURBS curve  $C_k^w(u)$  without sacrificing with the shape of the original section curve. This yields the common knot vector  $U$  in the parametric direction  $u$ , and  $\hat{n}'$  new control points  $Q_i^w$  on  $C_k^w(u)$ .
- (e) Make use of  $U$  and the values of the proposed parameterization method, as shown in Sect. 4.2; to compute the control points  $Q_i^w$  on  $C_k^w(u)$  for the cross-sectional curve with  $\hat{n}'$  data points.
- (f) Compute averaging parameter knot vector  $U_i$  of the  $i$ th cross-sectional curves with data points less than  $\hat{n}'$ . Then, compare  $U_i$  to  $U$ ; replacing knots of  $U_i$  with proximal knots of  $U$  yields knot vector  $\hat{U}_i$  (where the domain knots of  $\hat{U}_i$  should be part of the domain knot of  $U_i$ ).
- (g) Apply the knot refinement algorithm, as per Sect. 3, inserting  $(U - \hat{U}_i)$  knots in the  $\hat{U}_i$  to obtain the common knot vector  $U$  and to increase the  $i$ th cross-sectional curves data points to  $\hat{n}'$ .
- (h) Apply (e) to obtain  $\hat{n}'$  control points ( $Q_i^w$ ) on the NURBS curve  $C_k^w(u)$  of the  $i$ th cross-sectional curves.
- (i) Arrange the obtained control points ( $Q_i^w$ ) to form the control net of the resultant skinning surface as  $(Q_{k,l}^w)$ ,  $k = 0, 1, \dots, \hat{n}'$ ;  $l = 0, 1, \dots, K$ .
- (j) Use the control points  $Q_{k,l}^w$  to compute the averaging parameter knot vector  $V$  in the parametric direction  $v$ .
- (k) Generate the NURBS skinning surface using the control points  $Q_{k,l}^w$ , the common knot vector  $U$ , and the knot vector  $V$ .

#### 4.2 Proposed parameterization method

For any cross-sectional curve with data point  $P_i$ ,  $i = 0, 1, \dots, n$ , the parameter values will be computed as follows (rational/irrational B-spline basis functions B-spline basis functions can be used):

- (a) Using the centripetal method, choose an initial parameter value  $u_k$  and compute the average parameter knot vector  $U_{cp}$ .
- (b) Use  $u_k$  and  $U_{cp}$  to find the B-spline basis functions  $N_{i,p}(u_k)$  associated with each data point within each span index as follows:

$$N_{i,p}(u_k) = \frac{u_k - \lambda_i}{\lambda_{i+p} - \lambda_i} N_{i,p-1}(u_k) + \frac{\lambda_{i+p+1} - u_k}{\lambda_{i+p+1} - \lambda_{i+1}} N_{i-1,p-1}(u_k), \quad (15)$$

where

$$N_{i,0}(u_k) = \begin{cases} 1, & \text{if } \lambda_i \leq u_k < \lambda_{i+1}, \\ 0, & \text{otherwise.} \end{cases}$$

Assign  $N_k$  to the maximum B-spline basis function within the associated span index.

- (c) Find the rational B-splines basis functions  $R_{i,p}(u_k)$ , as given by Eq. 3, if rational B-spline basis functions will be used. Otherwise, go to the next step. Assign  $R_k$  to the maximum rational B-spline basis function within the associated span index.

- (d) Compute the square root of the absolute difference between each consecutive  $N_{k-1}$  and  $N_k$  or  $R_{k-1}$  and  $R_k$  as  $d_k$ , such that:

$$d_k = |N_k - N_{k-1}|^{1/2},$$

or

$$d_k = |R_k - R_{k-1}|^{1/2}, \quad k = 1, 2, \dots, n. \quad (16)$$

- (e) Compute the hybrid parameters value  $t_k$  as follows:

$$t_0 = 0, \quad t_n = 1$$

$$t_k = t_{k-1} + \frac{d_k}{D}, \quad k = 1, 2, \dots, n \quad (17)$$

where

$$D = \sum_{k=1}^n d_k. \quad (18)$$

The domain knots of averaging parameter knot vector  $U$  are computed using  $t_k$ , where each  $p$  consecutive parameter values from  $t_1$  to  $t_{n-1}$  are averaged as follows [21]:

$$u_{i+p} = \frac{1}{p} \sum_{k=1}^{i+p-1} t_k, \quad i = 1, 2, \dots, n-p. \quad (19)$$

The results of applying the proposed parameterization method and the resulting averaging parameter knot vectors which represent the cross-sectional curves of the forward perpendicular (FP) region and the resultant skinning surface show that either B-spline basis functions or rational B-spline basis functions are satisfying the property of “within each span index the rational/irrational B-spline basis functions are partition of unity”, as shown below.

$$N_k = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.00730.18140.50930.302 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.039 & 0.396 & 0.565 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.03490.502 & 0.441 & 0.022 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.03010.537 & 0.42710.0058 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.00070.29250.64650.0603 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.02030.62770.34410.0079 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.372 & 0.574 & 0.054 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.044 & 0.588 & 0.341 & 0.027 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.24370.54180.2145 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.17050.51930.30740.0028 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The results of applying the described NURBS skinning surface method, based on the proposed parameterization method for the FP region of an existing medium size oil tanker, are shown in the following section. For purposes of comparison, the same example was also addressed using conventional parameterization methods.

### 5 Results and discussion

The developed NURBS skinning approach was applied on the FP region of an existing medium size oil tanker. The FP region is composed of eight cross-sectional curves with data points ranging from 11 to 15.

First, the compatibility process was applied on these cross-sectional curves using four types of parameterization methods (uniform, chord length, centripetal, and the proposed method). Within an error tolerance 0.002-.15, the data of the cross-section with highest number of points was reduced, as shown in Table 1.

The results show that the proposed method and the chord length method give more data reduction than the other methods.

Cubic NURBS curves are generated for the compatible and non-compatible cross-sectional curves, as shown in Fig. 1. The results are based on the proposed method.

The cubic NURBS curves in Fig. 1a are generated over different knot vectors, while those in Fig. 1b are generated over the averaged parameter common knot vector  $U = \{0, 0, 0, 0, 0.2537, 0.313, 0.3566, 0.4202, 0.4655, 0.5249, 0.5988, 0.7733, 1, 1, 1, 1\}$ . The results show that within the error tolerance, the proposed method was capable of preserving the shapes of the original cross-sectional curves.

Data points of the compatible cross-sectional curves were arranged to form the resultant skinning surface. The results of our use of four parameterization methods were compared to approach [13]. The results are shown in Table 2.

The results indicate that our approach gave more data reduction on the resultant skinning surface. The best reductions were given by our proposed method and the chord length method.

The results of arranging the reduced data points of the cross-sectional curves gave  $(14 \times 8)$  control net for the uniform

method,  $(12 \times 8)$  control net for the chord length method,  $(13 \times 8)$  control net for the centripetal method, and  $(12 \times 8)$  control net for the proposed method. The common knot vectors  $U$  in the parametric direction  $u$ , and the knot vectors  $V$  in the parametric direction  $v$  for each case are as follows:

Uniform

$$U = \{0, 0, 0, 0, 0.0833, 0.1667, 0.25, 0.3333, 0.4167, 0.5833, 0.6667, 0.75, 0.8333, 0.9167, 1, 1, 1, 1\}$$

$$V = \{0, 0, 0, 0, 0.2, 0.4, 0.6, 0.8, 1, 1, 1, 1\}$$

Chord length

$$U = \{0, 0, 0, 0, 0.1659, 0.2037, 0.2904, 0.4278, 0.5102, 0.5925, 0.6434, 0.6982, 1, 1, 1, 1\}$$

$$V = \{0, 0, 0, 0, 0.2691, 0.3959, 0.5217, 0.6451, 1, 1, 1, 1\}$$

Centripetal

$$U = \{0, 0, 0, 0, 0.1268, 0.2195, 0.2571, 0.3031, 0.4229, 0.5058, 0.5881, 0.6558, 0.7089, 1, 1, 1, 1\}$$

$$V = \{0, 0, 0, 0, 0.2684, 0.395, 0.5204, 0.6433, 1, 1, 1, 1\}$$

Proposed

$$U = \{0, 0, 0, 0, 0.2537, 0.313, 0.3566, 0.4202, 0.4655, 0.5249, 0.5988, 0.7733, 1, 1, 1, 1\}$$

$$V = \{0, 0, 0, 0, 0.3447, 0.4349, 0.5546, 0.649, 1, 1, 1, 1\}$$

Based on these knot vectors and the reduced data points, bicubic NURBS skinning surfaces were generated for the FP region, as shown in Fig. 2.

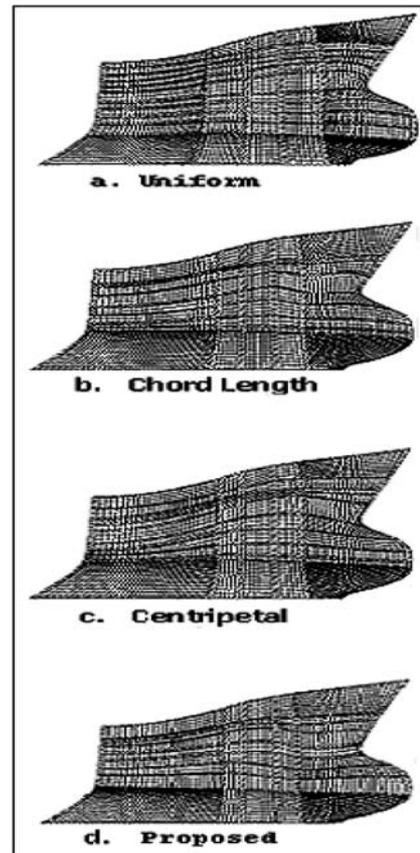
**Table 1.** Results of data reduction for the highest cross-section

Method	No. of data points	
	$\hat{n}$	$\hat{n}'$
Uniform	15	14
Chord length	15	12
Centripetal	15	13
Proposed	15	12

$\hat{n}$  : Highest index for the cross section with largest number of control points  
 $\hat{n}'$  : Index for reduced data point of the cross section with largest number of control points

**Table 2.** Results of data reduction for resultant skinning surface

		$K = 8$		Reduction rate %
		$\hat{n}$	$\hat{n}'$	
Hyungjun's approach		15	15	0
Developed approach	Uniform	15	14	6.67
	Chord length	15	12	20.00
	Centripetal	15	13	13.33
	Proposed	15	12	20.00



**Fig. 2.** Bicubic NURBS surfaces for the FP region

Figure 2 shows that the approach based on the proposed parameterization method reduced the number of data points by (20.0%) from the original cross-sectional curves and generates a better shape than the other three parameterization methods.

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## 6 Conclusion

This paper addresses the NURBS skinning surface problem in a set of open cross-sectional curves. A developed skinning NURBS approach based on a new parameterization method for ship hull design was proposed. Eight cross-sectional curves from the FP region of an existing medium size oil tanker with data points ranging from 11 to 15 were made compatible via the knot removal and knot refinement algorithms (compatibility process). Cubic NURBS curves were generated for the compatible and non-compatible cross-sectional curves. The results of the generated NURBS curves revealed the capability of the approach in reducing the number of data points while maintaining the original shape of the cross-sectional curves. The proposed parameterization method achieved a comparatively better reduction of the data points on the resultant skinning surface. Bicubic NURBS surfaces were generated for the resultant skinning surfaces via four types of parameterization. The results demonstrated the ability of our approach to generate smoother, better shapes in the FP region.

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